

変数分離形 次の微分方程式を解きなさい。また、一般解から得られない特異解がある場合にはそれも求めなさい。(M. Tennenbaum & H. Pollard: *Ordinary Differential Equations* (1963, 1985)より)

1. $y' = y$. 2. $x dy - y dx = 0$. 3. $\frac{dr}{d\theta} = -\sin \theta$.
4. $(y^2 + 1) dx - (x^2 + 1) dy = 0$. 5. $\frac{dr}{d\theta} \cot \theta - r = 2$.
6. $yx^2 dy - y^3 dx = 2x^2 dy$. 7. $(y^2 - 1) dx - (2y + xy) dy = 0$.
8. $x \log x dy + \sqrt{1 + y^2} dx = 0$. 9. $e^{x+1} \tan y dx + \cos y dy = 0$.
10. $x \cos y dx + x^2 \sin y dy = a^2 \sin y dy$.
11. $\frac{dr}{d\theta} = r \tan \theta$.
12. $(x - 1) \cos y dy = 2x \sin y dx$.
13. $y' = y \log y \cot x$.
14. $x dy + (1 + y^2) \operatorname{Arc} \tan y dx = 0$.
15. $dy + x(y + 1) dx = 0$.
16. $e^{y^2}(x^2 + 2x + 1) dx + (xy + y) dy = 0$.
17. $\frac{dy}{dx} + y = 0, \quad y(1) = 1$.
18. $\sin x \cos 2y dx + \cos x \sin 2y dy = 0, \quad y(0) = \pi/2$.
19. $(1 - x) dy = x(y + 1) dx, \quad y(0) = 0$.
20. $y dy + x dx = 3xy^2 dx, \quad y(2) = 1$.
21. $dy = e^{x+y} dx, \quad y(0) = 0$.
22. Define the differential of a function of three independent variables; of n independent variables.

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1. $y = ce^x$. 2. $y = cx$. 3. $r = \cos \theta + c$.
4. $\operatorname{Arc} \tan x = \operatorname{Arc} \tan y + c$ or $x - y = C(1 + xy)$, where $C = \tan c$.
5. $r = c \sec \theta - 2, \quad r \neq -2, \quad \theta \neq \frac{\pi}{2} + n\pi; \quad r = -2$.
6. $(cx + 1)y^2 = (y - 1)x, \quad x \neq 0, \quad y \neq 0; \quad y = 0$.
7. $x + 2 = c\sqrt{y^2 - 1}, \quad x \neq -2, \quad y \neq \pm 1; \quad y = \pm 1$.
8. $\log |x|(y + \sqrt{y^2 + 1}) = c, \quad x \neq 0, \quad x \neq 1$.
9. $e^{x+1} + \log (\csc y - \cot y) + \cos y = c, \quad y \neq n\pi; \quad y = n\pi$.
10. $a^2 - x^2 = c \cos^2 y, \quad x^2 \neq a^2, \quad y \neq \frac{\pi}{2} + n\pi; \quad y = \frac{\pi}{2} + n\pi$.
11. $r \cos \theta = c, \quad r \neq 0, \quad \theta \neq \frac{\pi}{2} + n\pi; \quad r = 0$.
12. $\sin y = (x - 1)^2 e^{2x+c}, \quad x \neq 1, \quad y \neq n\pi; \quad y = n\pi$.
13. $y = e^{c \sin x}, \quad x \neq n\pi, \quad y \neq 0$.
14. $y = \tan (c/x), \quad x \neq 0$.
15. $y = ce^{-x^2/2} - 1, \quad y \neq -1; \quad y = -1$.
16. $x^2 + 2x = e^{-y^2} + c, \quad x \neq -1$.
17. $y = e^{1-x}$.
18. $\cos^2 x \cos 2y = -1$.
19. $(y + 1)(1 - x) = e^{-x}$.
20. $3y^2 = 1 + 2e^{3x^2-12}$.
21. $e^x + e^{-y} = 2$.