

2階定数係数線形斉次微分方程式 次の微分方程式の一般解を求めなさい。31～35 については境界条件を満たす解を求めなさい。(M. Tennenbaum & H. Pollard: *Ordinary Differential Equations* (1963, 1985)より)

EXERCISE 20

Find the general solution of each of the following equations.

1. $y'' + 2y' = 0.$
2. $y'' - 3y' + 2y = 0.$
3. $y'' - y = 0.$
4. $y''' + y'' - 6y' = 0.$
9. $y^{(4)} + 4y''' + y'' - 4y' - 2y = 0.$
10. $y^{(4)} - a^2y = 0, \quad a > 0.$
11. $y'' - 2ky' - 2y = 0.$
12. $y'' + 4ky' - 12k^2y = 0.$
13. $y^{(4)} = 0.$
14. $y'' + 4y' + 4y = 0.$
20. $y^{(4)} + 2y''' - 11y'' - 12y' + 36y = 0.$
21. $36y^{(4)} - 37y'' + 4y' + 5y = 0.$
22. $y^{(4)} - 8y'' + 16y = 0.$
23. $y'' - 2y' + 5y = 0.$
24. $y'' - y' + y = 0.$
25. $y^{(4)} + 5y'' + 6y = 0.$
5. $6y'' - 11y' + 4y = 0.$
6. $y'' + 2y' - y = 0.$
7. $y''' + y'' - 10y' - 6y = 0.$
8. $y^{(4)} - y''' - 4y'' + 4y' = 0.$
15. $3y''' + 5y'' + y' - y = 0.$
16. $y''' - 6y'' + 12y' - 8y = 0.$
17. $y'' - 2ay' + a^2y = 0.$
18. $y^{(4)} + 3y''' = 0.$
19. $y^{(4)} - 2y'' = 0.$
26. $y'' - 4y' + 20y = 0.$
27. $y^{(4)} + 4y'' + 4y = 0.$
28. $y''' + 8y = 0.$
29. $y^{(4)} + 4y'' = 0.$
30. $y^{(5)} + 2y''' + y' = 0.$

For each of the following equations find a particular solution which satisfies the given initial conditions.

31. $y'' = 0, \quad y(1) = 2, \quad y'(1) = -1.$
32. $y'' + 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 1.$
33. $y'' - 2y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = 4.$
34. $y'' - 4y' + 20y = 0, \quad y(\pi/2) = 0, \quad y'(\pi/2) = 1.$
35. $3y''' + 5y'' + y' - y = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = -1.$

ANSWERS 20

1. $y = c_1 + c_2e^{-2x}.$
2. $y = c_1e^x + c_2e^{2x}.$
3. $y = c_1e^x + c_2e^{-x}.$
4. $y = c_1 + c_2e^{2x} + c_3e^{-3x}.$
5. $y = c_1e^{x/2} + c_2e^{4x/3}.$
6. $y = c_1e^{(-1+\sqrt{2})x} + c_2e^{(-1-\sqrt{2})x}.$

7. $y = c_1 e^{3x} + c_2 e^{(-2+\sqrt{2})x} + c_3 e^{(-2-\sqrt{2})x}.$
8. $y = c_1 + c_2 e^x + c_3 e^{2x} + c_4 e^{-2x}.$
9. $y = c_1 e^x + c_2 e^{-x} + c_3 e^{(-2+\sqrt{2})x} + c_4 e^{(-2-\sqrt{2})x}.$
10. $y = c_1 e^{\sqrt{a}x} + c_2 e^{-\sqrt{a}x} + c_3 \cos \sqrt{a} x + c_4 \sin \sqrt{a} x.$
11. $y = c_1 e^{(k+\sqrt{k^2+2})x} + c_2 e^{(k-\sqrt{k^2+2})x}.$
12. $y = c_1 e^{-6kx} + c_2 e^{2kx}.$
13. $y = c_1 + c_2 x + c_3 x^2 + c_4 x^3.$
14. $y = (c_1 + c_2 x) e^{-2x}.$
15. $y = c_1 e^{x/3} + (c_2 + c_3 x) e^{-x}.$
16. $y = (c_1 + c_2 x + c_3 x^2) e^{2x}.$
17. $y = (c_1 + c_2 x) e^{ax}.$
18. $y = c_1 + c_2 x + c_3 x^2 + c_4 e^{-3x}.$
19. $y = c_1 + c_2 x + c_3 e^{\sqrt{2}x} + c_4 e^{-\sqrt{2}x}.$
20. $y = (c_1 + c_2 x) e^{2x} + (c_3 + c_4 x) e^{-3x}.$
21. $y = c_1 e^{-x} + c_2 e^{x/2} + c_3 e^{5x/6} + c_4 e^{-x/3}.$
22. $y = (c_1 + c_2 x) e^{2x} + (c_3 + c_4 x) e^{-2x}.$
23. $y = e^x (c_1 \cos 2x + c_2 \sin 2x).$
24. $y = c_1 e^{\frac{1+\sqrt{3}i}{2}x} + c_2 e^{\frac{1-\sqrt{3}i}{2}x} = e^{x/2} \left(c_3 \cos \frac{\sqrt{3}}{2} x + c_4 \sin \frac{\sqrt{3}}{2} x \right).$
25. $y = c_1 \cos \sqrt{3} x + c_2 \sin \sqrt{3} x + c_3 \cos \sqrt{2} x + c_4 \sin \sqrt{2} x.$
26. $y = e^{2x} (c_1 \cos 4x + c_2 \sin 4x).$
27. $y = (c_1 + c_2 x) \cos \sqrt{2} x + (c_3 + c_4 x) \sin \sqrt{2} x.$
28. $y = c_1 e^{-2x} + e^x [c_2 \cos \sqrt{3} x + c_3 \sin \sqrt{3} x].$
29. $y = c_1 + c_2 x + c_3 \cos 2x + c_4 \sin 2x.$
30. $y = c_1 + c_2 \sin x + c_3 \cos x + c_4 x \sin x + c_5 x \cos x.$
31. $y = 3 - x.$
32. $y = (1 + 3x) e^{-2x}.$
33. $y = e^x (2 \cos 2x + \sin 2x).$
34. $y = \frac{1}{4} e^{2x-\pi} \sin 4x.$
35. $y = \frac{9}{16} e^{x/3} + \left(\frac{x}{4} - \frac{9}{16} \right) e^{-x}.$